Four-neutrino model and the K2K experiment

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Abstract

We investigate the neutrino oscillations of ν_μ beam at the K2K experiment in the four-neutrino model with three active and one sterile neutrinos, and compare them with the oscillations in the three-neutrino model. In the four-neutrino case, the effect of the $\Delta m^2_{\rm LSND}$ scale of mass-squared difference, derived from the LSND experiments, occurs in the survival probability $P(\nu_\mu \to \nu_\mu)$ in the range of $\Delta m^2 < 0.004$ eV^2, where Δm^2 is the relevant one to the K2K experiment and corresponds to the atmospheric neutrino mass scale. Once the probability $P(\nu_\mu \to \nu_\mu)$ is measured at the K2K, the allowed region of Δm^2 would turn out to be broader in the four-neutrino model than the one in the three-neutrino model.

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It has turned out that neutrinos have a certain amount of mass through the observations of the atmospheric neutrino anomaly[1] [2]. The anomaly can naturally be explained by the neutrino oscillation[3], along with the analyses of the solar neutrino deficit[4]. The oscillation, however, gives only the mass-squared difference among various species of neutrinos.

At present, if the LSND experiment is included, three kinds of mass-squared differences are derived: $\Delta m^2_{\rm solar} = (10^{-11} - 10^{-5})\rm eV^2$ from the solar neutrino deficit with a large range of Δm^2, depending on the four solutions of the vacuum oscillation, and the Mikheyev-Smirnov-Wolfenstein (MSW) solutions in the matter with small- and large-angle mixings and the LOW one with relatively low mass-squared difference[5], $\Delta m^2_{\rm atm} = (1.5 - 5) \times 10^{-3}\ \rm eV^2$ from the atmospheric neutrino anomaly with a large mixing angle of $\sin^2 2\theta_{\rm atm} > 0.82$ interpreted as the $\nu_\mu \to \nu_\tau$ oscillation, and $\Delta m^2_{\rm LSND} = (0.2 - 2)\rm eV^2$ from the LSND experiments on $\nu_\mu \to \nu_e$ and $\bar{\nu_\mu} \to \bar{\nu_e}$ oscillations[6], which is the only one positive evidence from the terrestrial oscillation experiments using the accelerators and reactors.

It is eagerly desired to know the magnitude of mass itself, which would be given by the neutrinoless double beta decays. It is, however, impossible to know the neutrino masses at present. So, for the moment, it is important to pinpoint some of the mass-squared differences to the required accuracy by doing various experiments. One of the experiments, which is now running, is the K2K experiment[7].

In this paper, we investigate the oscillations of the ν_μ beam, which is being measured at the K2K, in the four-neutrino model with mass scheme of the two nearly degenerate pairs separated by the order of 1eV for the three active and one sterile neutrinos[8-15] by using the constraints on the mixing matrix derived from the solar neutrino deficit, atmospheric neutrino anomaly, Bugey reactor experiment, CHOOZ experiment, LSND experiments, CHORUS and NOMAD experiments and the other accelerator and reactor experiments. And, we compare the

oscillations with the ones in the three-neutrino model.

Under the neutrino oscillation hypothesis[16][17], the flavor eigenstates are the mixtures of mass eigenstates with mass m_i $(i = 1, 2, 3, 4)$ in the four-neutrino model as follows:

$$\nu_\alpha = \sum_{i=1}^{4} U_{\alpha i}\nu_i, \qquad \alpha = e, \mu, \tau, s \tag{1}$$

where ν_e, ν_μ and ν_τ are the ordinary neutrinos and ν_s is the sterile one, and U is the unitary mixing matrix. The neutrino oscillation probability of $\nu_\alpha \to \nu_\beta$ in vacuum is given in the usual manner by

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathrm{Re}(U^*_{\alpha k}U_{\alpha j}U^*_{\beta j}U_{\beta k}) \sin^2\Delta_{kj} + 2\sum_{k>j} \mathrm{Im}(U^*_{\alpha k}U_{\alpha j}U^*_{\beta j}U_{\beta k}) \sin 2\Delta_{kj}, \tag{2}$$

where $\Delta_{kj} \equiv \Delta m^2_{kj}L/(4E)$, L being the distance from the neutrino source and E the energy of neutrino.

The four neutrino masses should be devided into two pairs of close masses separated by a gap of about 1eV in order to accomodate with the solar and atmospheric neutrino deficits and the LSND experiments along with the other results from the accelerator and reactor experiments on the neutrino oscillation. There are the following two schemes for the mass pattern; (i) $\Delta m^2_{\rm solar} \equiv \Delta m^2_{21} \ll \Delta m^2_{\rm atm} \equiv \Delta m^2_{43} \ll \Delta m^2_{\rm LSND} \equiv \Delta m^2_{32}$, and (ii) $\Delta m^2_{\rm solar} \equiv \Delta m^2_{43} \ll \Delta m^2_{\rm atm} \equiv \Delta m^2_{21} \ll \Delta m^2_{\rm LSND} \equiv \Delta m^2_{32}$, where $\Delta m^2_{kj} \equiv m^2_k - m^2_j$. We will adopt the first scheme in the following analyses, and the second scheme can be attained only through the exchange of indices $(1, 2) \leftrightarrow (3, 4)$ in the following various expressions such as the oscillation probabilities. The constraints on the mixing matrix U are derived in the four-neutrino model in the following[18].

(i) Solar neutrino deficit. Since $\Delta_{21} \sim 1$ and all the other five Δ_{kj}'s are enormously larger than 1, the survival probability of ν_e is given from Eq.(2) by

$$P_{\rm solar}(\nu_e \to \nu_e) \;\; \simeq \;\; 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\Delta_{21} - 2|U_{e3}|^2(1 - |U_{e3}|^2 - |U_{e4}|^2)$$

$$-2|U_{e4}|^2(1-|U_{e4}|^2), \tag{3}$$

where the unitarity of U is used. For the solar neutrino deficit, there are four different kinds of solutions as stated above, and a unique solution is not yet found, so that we will not use this deficit in order to obtain the constraints on U.

(ii) Atmospheric neutrino anomaly. Since $\Delta_{21} \ll 1, \Delta_{43} \sim 1$ and $\Delta_{41}, \Delta_{42}, \Delta_{31}, \Delta_{32} \gg 1$, the survival probability of ν_μ is given by

$$P_{\rm atm}(\nu_\mu \to \nu_\mu) \simeq 1-4|U_{\mu 3}|^2|U_{\mu 4}|^2 \sin^2\Delta_{43} - 2(|U_{\mu 1}|^2+|U_{\mu 2}|^2)(1-|U_{\mu 1}|^2-|U_{\mu 2}|^2). \tag{4}$$

By using the data from the Super-Kamiokande experiments, $\sin^2 2\theta_{\rm atm} > 0.82$ for $5\times10^{-4} < \Delta m^2 < 6\times10^{-3}$ eV^2, and expecting from this data that $|U_{\mu 1}|^2+|U_{\mu 2}|^2 \ll 1$, the following constraint is obtained,

$$|U_{\mu 3}|^2|U_{\mu 4}|^2 > 0.205. \tag{5}$$

(iii) The Bugey experiment[19] (including Krasnoyarsk[20], CDHS[21] and CCFR[22] experiments). By being typically represented by the Bugey reactor experiment with $L/E = 3-20$ [m/MeV], since $\Delta_{21} \ll 1, \Delta_{43} \ll 1$ and $\Delta_{41}, \Delta_{42}, \Delta_{31}, \Delta_{32} \sim 1$, the survival probability of $\bar{\nu}_e$ is given by

$$P_{\rm Bugey}(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1-4(|U_{e3}|^2+|U_{e4}|^2)(1-|U_{e3}|^2-|U_{e4}|^2)\sin^2\Delta_{32}. \tag{6}$$

If we use the data from the Bugey experiment conservatively, $\sin^2 2\theta_{\rm Bugey} < 0.1$ for $0.1 < \Delta m^2 < 1$ eV^2, the following constraint is obtained:

$$|U_{e3}|^2+|U_{e4}|^2 < 0.025. \tag{7}$$

The first long-baseline reactor experiment, that is, the CHOOZ experiment [23] with $L/E \sim 300$ [km/GeV] gives a constraint of $4|U_{e3}|^2|U_{e4}|^2 < 0.18$ through their data of $\sin^2 2\theta_{\rm CHOOZ} < 0.18$ for $3\times10^{-3} < \Delta m^2 < 1.0\times10^{-2}$ eV^2. However, this constraint can be included in the constraint of Eq.(7) from the Bugey experiment.

In the same way as the above, the LSND experiments[6] with $L/E = 0.5 - 1$ [m/MeV] brings the constraint of

$$|U^*_{\mu 3} U_{e3} + U^*_{\mu 4} U_{e4}| = 0.02 - 0.16 \tag{8}$$

from the data of $\sin^2 2\theta_{\rm LSND} = 1.5\times 10^{-3} - 1.0\times 10^{-1}$ for $0.2 < \Delta m^2 < 2\ {\rm eV}^2$. And, CHORUS[24] and NOMAD[25] experiments searching for the $\nu_\mu \to \nu_\tau$ oscillation with $L/E = 0.02 - 0.03$ [km/GeV] give the constraint of

$$|U^*_{\mu 3} U_{\tau 3} + U^*_{\mu 4} U_{\tau 4}| < 0.28 \tag{9}$$

from the data of $\sin^2 2\theta_{\rm NOMAD} < 0.3$ for $\Delta m^2 < 2.2{\rm eV}^2$. Therefore, among the abovementioned six typical phenomena and experiments, the useful constraints are of Eqs. (5), (7), (8) and (9).

In order to translate these four constraints to the ones for the mixing angles and phases, we adopt the most general parametrization of the mixing matrix for Majorana neutrinos, proposed by Barger, Dai, Whisnant and Young [15], which includes six mixing angles and six phases. The expression of the matrix is too complicated to write it down here, so that we cite only the matrix elements which are useful for the following analyses; $U_{e1} = c_{01}c_{02}c_{03}$, $U_{e2} = c_{02}c_{03}s^*_{d01}$, $U_{e3} = c_{03}s^*_{d02}$, $U_{e4} = s^*_{d03}$, $U_{\mu 3} = -s^*_{d02}s_{d03}s^*_{d13} + c_{02}c_{13}s^*_{d12}$, $U_{\mu 4} = c_{03}s^*_{d13}$, $U_{\tau 3} = -c_{13}s^*_{d02}s_{d03}s^*_{d23} - c_{02}s^*_{d12}s_{d13}s^*_{d23} + c_{02}c_{12}c_{23}$, and $U_{\tau 4} = c_{03}c_{13}s^*_{d23}$, where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{dij} \equiv s_{ij}{\rm e}^{{\rm i}\delta_{ij}} \equiv \sin\theta_{ij}{\rm e}^{{\rm i}\delta_{ij}}$ [15], and θ_{01}, θ_{02}, θ_{03}, θ_{12}, θ_{13}, θ_{23} are the six angles and δ_{01}, δ_{02}, δ_{03}, δ_{12}, δ_{13}, δ_{23} are the six phases. Three of the six oscillation probability differences are independent so that only three of the six phases determine the oscillation probabilities, that is, the Dirac phases.

By using this parametrization of U, the four constraints of Eqs.(5), (7), (8) and (9) are expressed by the angles and phases as follows:

$$|-s_{02}s_{03}s_{13}{\rm e}^{-{\rm i}(\delta_{02}-\delta_{03}+\delta_{13})} + c_{02}c_{13}s_{12}{\rm e}^{-{\rm i}\delta_{12}}|^2 c_{03}^2 s_{13}^2 > 0.205, \tag{10}$$

$$c_{03}^2 s_{02}^2 + s_{03}^2 < 0.025, \tag{11}$$

$$|c_{02}s_{02}c_{03}s_{12}c_{13} + c_{02}^2 c_{03}s_{03}s_{13}\mathrm{e}^{\mathrm{i}\delta_1}| = 0.02 - 0.16, \tag{12}$$

$$\begin{aligned} | \quad & c_{02}^2 c_{12}s_{12}c_{13}c_{23} - c_{02}s_{02}s_{03}s_{12}c_{13}^2 s_{23}\mathrm{e}^{-\mathrm{i}(\delta_1+\delta_2)} - c_{02}s_{02}s_{03}c_{12}s_{13}c_{23}\mathrm{e}^{\mathrm{i}\delta_1} \\ & + c_{13}s_{13}s_{23}(c_{03}^2 - c_{02}^2 s_{12}^2 + s_{02}^2 s_{03}^2)\mathrm{e}^{-\mathrm{i}\delta_2}| < 0.28, \end{aligned} \tag{13}$$

where $\delta_1 \equiv \delta_{02} - \delta_{03} - \delta_{12} + \delta_{13}$ and $\delta_2 \equiv \delta_{12} - \delta_{13} + \delta_{23}$. The constraint of Eq.(10) reduces to

$$s_{12}^2 c_{13}^2 s_{13}^2 > 0.205 \tag{14}$$

due to the smallness of s_{02} and s_{03}, which is obtained from Eq.(11). By using the constraints of Eqs.(11) and (14) together with the nearly maximal mixing in the angle θ_{23}, which is derived from the large angle mixing in $\nu_\mu \to \nu_\tau$ oscillation for the atmospheric neutrino anomaly, it proves that no constraints on the phases δ_1 and δ_2 are obtained from Eqs.(12) and (13). Equation (12), however, gives a constraint on the mixing angles. So, we obtain three constraints of Eqs.(11), (12) and (14) on the mixing angles and no constraints on the two phases of δ_1 and δ_2 in the four-neutrino model. The third Dirac phase does not give any significant effect to the leading parts of the oscillation probabilities, as can be seen from its no occurence in the constraints of Eqs.(10) – (13).

By using the constraints of Eqs.(11), (12) and (14), we calculate the oscillation probabilities of muon neutrino for the K2K experiment. We set the distance between the neutrino detector and the source L to be 250 km and the energy of neutrino E to be 1.4 GeV. A typical result is shown in Fig.1, which gives the survival probability $P(\nu_\mu \to \nu_\mu)$ and disappearance probabilities of $P(\nu_\mu \to \nu_\tau), P(\nu_\mu \to \nu_e)$, and $P(\nu_\mu \to \nu_s)$ with respect to Δm_{43}^2, which corresponds to $\Delta m_{\mathrm{atm}}^2$. We took the parameters as $s_{01} = s_{23} = 1/\sqrt{2}, s_{02} = s_{03} = 0.11, s_{12} = 0.91, s_{13} = 0.67$, and $\delta_1 = \delta_2 = \pi/2$. Here and in the following, we take $\Delta m_{21}^2 (\equiv \Delta m_{\mathrm{solar}}^2) = 1.0 \times 10^{-6}$ eV^2 and $\Delta m_{32}^2 (\equiv \Delta m_{\mathrm{LSND}}^2) = 0.3$ eV^2. The difference of

$P(\nu_\mu \to \nu_\mu)$ of 0.7 from 1 around $0.0001 \le \Delta m^2_{43} \le 0.001$ eV^2 comes from the $\Delta m^2_{\rm LSND}$ contribution, as pointed out by Yasuda [26]. It goes up to $1.0 - 0.96$ in the same region of Δm^2_{43}, as the angle s_{12} is increased towards 1.0. As seen in Fig.1, $P(\nu_\mu \to \nu_\mu)$ does not vary in $0.0001 \le \Delta m^2_{43} \le 0.001$ eV^2 and decreases abruptly from 0.6 to 0.07 in the region of $0.002 \le \Delta m^2_{43} \le 0.006$ eV^2. $P(\nu_\mu \to \nu_\tau)$ and $P(\nu_\mu \to \nu_s)$ are very small in $0.0001 \le \Delta m^2_{43} \le 0.001$ eV^2 and take a sizable magnitude of $0.3 - 0.6$ in $\Delta m^2_{43} = 0.004 - 0.006$ eV^2. In the computation we have not convoluted the probabilities with respect to the energy spread of the incident neutrinos. The feature of the computational results is as follows: (i) The dependence of the oscillation probabilities on the phases δ_1 and δ_2 is very weak. (ii) The change of the probabilities between $s_{02} = s_{03} = 0.11$ and 0.05 is very small for $P(\nu_\mu \to \nu_\mu), P(\nu_\mu \to \nu_\tau)$ and $P(\nu_\mu \to \nu_s)$, while the change is not so small for $P(\nu_\mu \to \nu_e)$ since the mixing angles s_{02} and s_{03} affect significantly the $\nu_\mu \to \nu_e$ oscillation. (iii) The dependence on the sign of the mixing angles s_{12} and s_{13} is that $P(\nu_\mu \to \nu_\mu)$ is same between the cases of $s_{12} > 0, s_{13} > 0$ and $s_{12} > 0, s_{13} < 0$, and $P(\nu_\mu \to \nu_\tau)$ and $P(\nu_\mu \to \nu_s)$ interchange with each other between the two cases, and that all the probabilities do not change between the cases of $s_{12} > 0, s_{13} > 0$ and $s_{12} < 0, s_{13} < 0$ and also the same between the cases of $s_{12} > 0, s_{13} < 0$ and $s_{12} < 0, s_{13} > 0$.

Next, we will compare these results with those in the three-neutrino model with three active neutrinos, where the mass pattern is taken as $\Delta m^2_{21} = \Delta m^2_{\rm solar}$ and $\Delta m^2_{32} = \Delta m^2_{\rm atm}$. The constraints on the mixing matrix are derived as follows. The solar neutrino deficit is not used again here, since there are so many (four) solutions.

(i) Atmospheric neutrino anomaly. The survival probability of ν_μ is given by

$$P_{\rm atm}(\nu_\mu \to \nu_\mu) \simeq 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)\sin^2\Delta_{32}. \tag{15}$$

Then, the constraint is replaced by $|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) > 0.205$, instead of Eq.(5).

This gives us the following constraint on $|U_{\mu 3}|$,

$$0.54 < |U_{\mu 3}| < 0.84. \tag{16}$$

(ii) The CHOOZ experiment. The survival probability of $\bar{\nu_e}$ is given by

$$P_{\rm CHOOZ}(\bar{\nu_e} \to \bar{\nu_e}) \simeq 1 - 4|U_{e3}|^2(1 - |U_{e3}|^2)\sin^2\Delta_{32}. \tag{17}$$

The data stated in the description of the four-neutrino model gives the constraint of $4|U_{e3}|^2(1 - |U_{e3}|^2) < 0.18$, leading to the following constraint on $|U_{e3}|$,

$$|U_{e3}| < 0.22. \tag{18}$$

In the three-neutrino model, the constraint from the Bugey experiment can be included in the constraint of Eq.(18) from the CHOOZ experiment.

(iii) The CHORUS and NOMAD experiments. The transition probability of $\nu_\mu \to \nu_\tau$ is given by

$$P(\nu_\mu \to \nu_\tau) \simeq 4|U_{\mu 3}|^2|U_{\tau 3}|^2\sin^2\Delta_{32}. \tag{19}$$

Their data of $\sin^2 2\theta_{\rm NOMAD} \leq 1$ for $\Delta m^2 = (1.5 - 5)\times 10^{-3}$ eV^2 gives a constraint of

$$|U_{\mu 3}||U_{\tau 3}| < 0.5. \tag{20}$$

In order to translate these three constraints of Eqs.(16), (18) and (20) to the ones for the mixing angles and phases, we choose the parametrization used by the PDG[27] with the additional two phases for Majorana neutrinos[28], which eventually includes three angles and three phases. We give here only the relevant elements; $U_{e3} = s_{13}{\rm e}^{{\rm i}(\rho-\phi)}, U_{\mu 3} = s_{23}c_{13}{\rm e}^{{\rm i}(\rho-\beta)}$, and $U_{\tau 3} = c_{23}c_{13}$, where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ and ρ, ϕ and β are the three phases. As in the four-neutrino model, the Majorana phases ρ and β do not enter into the oscillation probabilities. The three constraints are expressed with the three angles as

$$0.54 \;<\; |s_{23}c_{13}| < 0.84, \tag{21}$$

$$|s_{13}| \; < \; 0.22, \qquad (22)$$

$$c_{13}^2|c_{23}s_{23}| \; \leq \; 0.5. \qquad (23)$$

The constraint of Eq.(23) is always satisfied for any values of s_{23} and s_{13}. So, we have two constraints of Eqs.(21) and (22) remained. We again calculate the oscillation probabilities of the ν_μ beam for the K2K experiment. A typical result is shown in Fig.2 for $P(\nu_\mu \to \nu_\mu), P(\nu_\mu \to \nu_\tau)$, and $P(\nu_\mu \to \nu_e)$ with respect to $\Delta m_{32}^2(= \Delta m_{\rm atm}^2)$, where we took $s_{12} = s_{23} = 1/\sqrt{2}, s_{13} = 0.15$, and $\phi = \pi/2$. The probability $P(\nu_\mu \to \nu_\mu)$ is nearly $0.95 - 1$ in $0.0001 \leq \Delta m_{32}^2 \leq 0.001$ eV^2 and decreases rapidly from 0.8 to 0.04 in the region of $0.002 \leq \Delta m_{32}^2 \leq 0.006$ eV^2. $P(\nu_\mu \to \nu_\tau)$ takes a sizable magnitude of $0.2 - 0.6$ around $\Delta m_{32}^2 = 0.002 - 0.004$ eV^2 and $P(\nu_\mu \to \nu_e)$ is quite small due to the small mixing angle s_{13}, which comes from the constraint by the CHOOZ experiment. The dependence of the probabilities on the phase ϕ is very weak.

Fig.3 shows the regions of $P(\nu_\mu \to \nu_\mu)$ allowed by the constraints on the mixing angles and phases discussed above in the three-neutrino model(dashed lines) and in the four-neutrino model(solid lines). The allowed region is the one sandwiched between the upper curve(maximum) and the lower one(minimum). It is very broad in the four-neutrino model, especially in the range of $0.0001 \leq \Delta m^2 \leq 0.002$ eV^2 due to the range of $0.91 \leq s_{12} \leq 1$ derived from the constraint of Eq.(14), where the minimum curve of $P(\nu_\mu \to \nu_\mu)$ is given by $s_{12} = 0.91$ and the maximum one is by $s_{12} = 1.0$. On the contrast, the region of $P(\nu_\mu \to \nu_\mu)$ is considerably limited in the three-neutrino model, determined by the range of $0.55 \leq s_{23} \leq 0.85$ coming from the constraint of Eq.(21). The minimum curve is given by $s_{23} = 1/\sqrt{2}$ and the maximum one is by $s_{23} = 0.55$(or 0.85), where the maximum curve almost overlaps with the one in the four-neutrino model. The two models do not show any significant difference to $P(\nu_\mu \to \nu_\mu)$ in the range of $0.004 \leq \Delta m^2 \leq 0.007$ eV^2, while they give a large difference for $\Delta m^2 < 0.004$ eV^2. For example, if the

probability $P(\nu_\mu \to \nu_\mu)$ is measured to be 0.6 by the K2K experiment, the three-neutrino model will predict that the relevant scale of mass-squared difference is $0.0030 \leq \Delta m^2 \leq 0.0034$ eV^2, a relatively narrow range, while the four-neutrino model will predict that it is $0.0020 \leq \Delta m^2 \leq 0.0034$ eV^2, a broader range than in the three-neutrino model. That is to say, the determination of the relevant scale of Δm^2 by the K2K experiment is more flexible in the four-neutrino model than in the three-neutrino model. That is a big advantage at the time when $\Delta m^2_{\rm atm}$ is not so precisely determined by the experiments and/or the observations, for example, of the atmospheric neutrino deficit.

A way to predict more precisely the allowed region of $P(\nu_\mu \to \nu_\mu)$ for the K2K experiment in the four-neutrino model is to determine the angle s_{12} to a good precision, which will be attained by the CHORUS/NOMAD-type experiment for the $\nu_\mu \to \nu_\tau$ oscillation performed with a longer distance $L \simeq 30$ km so that L/E becomes $\simeq 1$, since $P(\nu_\mu \to \nu_\tau)$ is predominantly controlled by the first term with the angle s_{12} in the left-hand side of Eq.(13).

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Figure captions

Fig.1. The probabilities $P(\nu_\mu \to \nu_\mu)$ (solid line), $P(\nu_\mu \to \nu_\tau)$ (dash-dotted line), $P(\nu_\mu \to \nu_s)$ (dashed line), and $P(\nu_\mu \to \nu_e)$ (dotted line) versus Δm^2_{43} calculated in the four-neutrino model for the K2K experiment with $L = 250$ km and $E = 1.4$ GeV. The parameter values of the mixing angles and phases are $s_{01} = s_{23} = 1/\sqrt{2}, s_{02} = s_{03} = 0.11, s_{12} = 0.91, s_{13} = 0.67$, and $\delta_1 = \delta_2 = \pi/2$.

Fig.2. The probabilities $P(\nu_\mu \to \nu_\mu)$ (solid line), $P(\nu_\mu \to \nu_\tau)$ (dash-dotted line), and $P(\nu_\mu \to \nu_e)$ (dotted line) versus Δm^2_{32} calculated in the three-neutrino model for the K2K experiment. The parameter values of the mixing angles and phases are $s_{12} = s_{23} = 1/\sqrt{2}, s_{13} = 0.15$, and $\phi = \pi/2$.

Fig.3. The regions of $P(\nu_\mu \to \nu_\mu)$ for the K2K experiment allowed by the constraints on the mixing angles and phases in the four-neutrino model (solid lines) and in the three-neutrino model (dashed lines). The allowed regions are in between the upper and lower curves. The upper curve in the three-neutrino model almost overlaps with the upper curve in the four-neutrino model. The parameter values are $s_{01} = s_{23} = 1/\sqrt{2}, s_{02} = 0.024, s_{03} = 0.0, s_{12} = 1.0, s_{13} = 0.54$ and $\delta_1 = \delta_2 = \pi/2$ for the upper curve, and $s_{01} = s_{23} = 1/\sqrt{2}, s_{02} = s_{03} = 0.11, s_{12} = 0.91, s_{13} = 0.67$ and $\delta_1 = 0, \delta_2 = \pi/2$ for the lower curve in the four-neutrino model. $s_{12} = 1/\sqrt{2}, s_{23} = 0.55, s_{13} = 0.20$ and $\phi = \pi/2$ for the upper curve, and $s_{12} = s_{23} = 1/\sqrt{2}, s_{13} = 0.20$ and $\phi = \pi/2$ for the lower curve in the three-neutrino model.

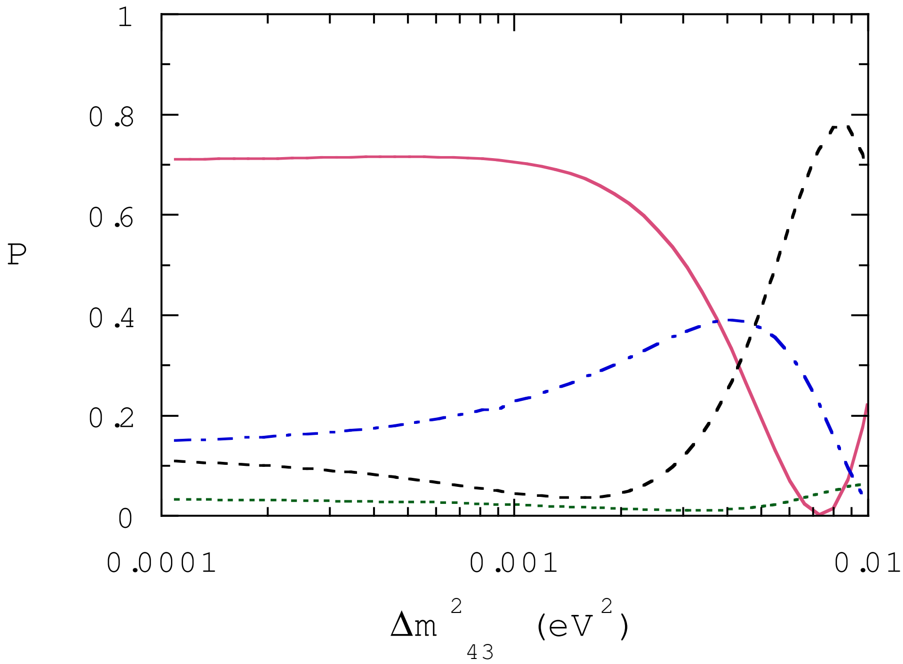

Fig.1

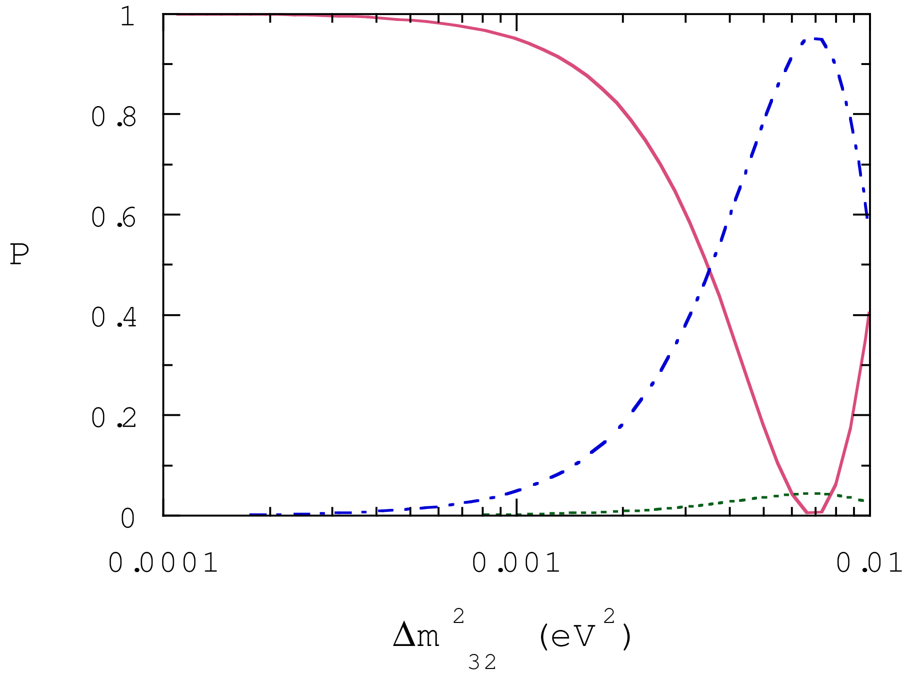

Fig.2

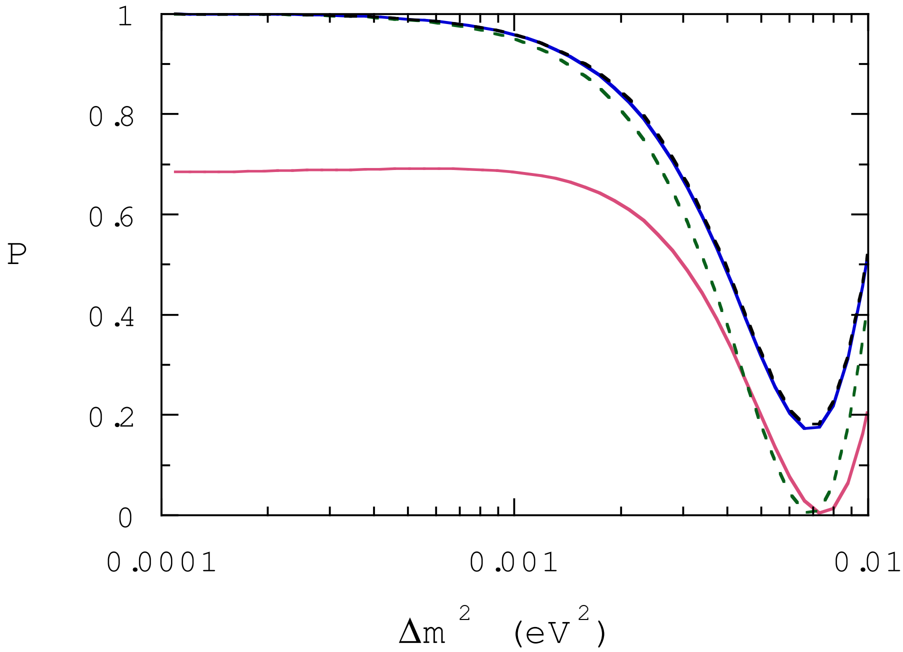

Fig.3